An executive summary of the final report of work done on the minor research project of **John Sherra** entitled

"Unique Metro Domination"

sanctioned by UGC vide sanction letter No.MRP(S)-0123/12-13/KAMA002/UGC-SWRO dated 23rd September 2013

A metro dominating set D of a graph G(V; E) is called a unique metro dominating set (in short an UMD-set) if |N(v) ∩ D| = 1,for each vertex v ∈ V – D. A UMD-set of G is called a minimal unique metro domination number of G if none of its proper subsets is an UMD-set for G. The minimum cardinality of a minimal UMD-set of a graph G is called Unique metro domination number or Uni-Metro domination number and is denoted by $\gamma_{\mu\beta}$ (G).

Complete Graphs were studied and it was found out that $\gamma_{\mu\beta}\,(K_n) = \begin{cases} 1, if\ n = 1\ or\ 2 \\ n, otherwise \end{cases}$

The study in paths P_n revealed the result,

$\gamma_{\mu\beta}$ (P_1)=1, $\gamma_{\mu\beta}$ (P_2)=1, $\gamma_{\mu\beta}$ (P_3)=2 and $\gamma_{\mu\beta}$ (P_n)=$\left\lfloor\frac{n+2}{3}\right\rfloor$for all n >3.

In cycles C_n, it was proved that order of the gap between any two vertices in the unique metro dominating set is at most two. Using this basic result the following results were proved in Cycles.

1) If D is a dominating set of C_n then $|D| \geq \frac{n}{3}$
2) For any i, the vertices v_i and v_{i+3} resolve the vertex set V of C_n.
3) If D is a UMD set for C_n then the gaps between the vertices must be of order 2 or of order 0.
4) For any integer k≥3, $\gamma_{\mu\beta}$ (C_{3k})=k
5) For any integer k≥2, $\gamma_{\mu\beta}$ (C_{3k+1})=k+1
6) For any integer k≥2, $\gamma_{\mu\beta}$ (C_{3k+2})=k+2

Summarizing these, we get then theorem,

$\gamma_{\mu\beta}$ (C_n)$= 3\ if\ n = 3,5.$ $\gamma_{\mu\beta}$ (C_n)$= 2\ if\ n = 4.$ $\gamma_{\mu\beta}$ (C_n)=4 if n=6.

$$\gamma_{\mu\beta}(C_n)=\begin{cases} \left\lceil\frac{n}{3}\right\rceil if\ n \equiv 0,1\ (mod3) and\ n \geq 7 \\ \left\lceil\frac{n}{3}\right\rceil + 1\ if\ n \equiv 2\ (mod3) and\ n \geq 7 \end{cases}$$

It is also proved that for given positive integers with $3 \leq m \leq n \leq 4m, there\ is\ a\ graph\ G\ with\ n\ vertices\ such\ that\ \gamma_{\mu\beta}$ (G)is m.

Cartesian product of two graphs is defined. As P_nXP_2 has 2n vertices a dominating set of P_nXP_2, D should satisfy, $|D| \geq \frac{n}{2}$. In order to reduce |D| we have to increase the order of the gaps. Most suitable gaps are of the order 3. It is also justified that atleast one of $\{v_{1,1}, v_{1,2}, v_{2,1}, v_{2,2}\}$ must be in D.

Suppose $v_{1,1}$ is in D_1 and $v_{1,2}$ is in D_2 then $|D_1| \leq |D_2|$. It is proved that D contains $v_{1,1+4i}$, $0 \leq 4i \leq n$ and $v_{2,\ 3+4j}$, $0 \leq 3+4j \leq n$. Using these results the following lemmas are proved.

When n = 1+4k. |D| = $\frac{n+1}{2}$. When n = 4k+3, |D| = $\frac{n+1}{2}$.

When n = 4k+2, |D| = $\frac{n}{2} + 1$. When n = 4k, |D| = $\frac{n}{2} + 1$.

Thus we obtain the result $\gamma(P_n \text{ X } P_2) = \left\lfloor\frac{n+2}{2}\right\rfloor$.

For n > 4, it is proved that the vertices $v_{1,1}$, $v_{2,3}$ and $v_{1,5}$ resolve all the vertices of V – D. When n=4 it was found out that the vertices $v_{1,1}$ and $v_{1,3}$ resolve all vertices of V – D.

For Unique domination 4 different cases were studied. When $v_{1,j}$ and $v_{1,\ j+4}$ are in D, $v_{2,\ j+2}$ is also in D. The vertices in th gap $v_{1,\ j+1}$, $v_{1,\ j+2}$ and $v_{1,\ j+2}$ are uniquely dominated. Further $v_{2,1}$ and $v_{2,\ 2}$ are also uniquely dominated.

Thus the theorem,

$\gamma_{\mu\beta}(P_n \text{ X } P_2) = \left\lfloor\frac{n+2}{2}\right\rfloor$, for any n≥4 is proved.

Date: 16-11-2015 **John Sherra**